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COMMENT

# Comments on the electromagnetic fields of a time-dependent solenoid

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**Abstract.** The electromagnetic fields associated with a time-dependent infinitely long solenoid are analysed. It is proved that if the magnetic field vanishes at the solenoidal outer region then its flux is a linear function of the time. It is also shown that a time-dependent solenoid does not satisfy the field-free requirement of the Aharonov-Bohm effect.

An infinite solenoid is frequently used in a theoretical analysis of electromagnetic interactions. A solenoid with a time-dependent flux belongs to this category. The electromagnetic potential and fields of this system are the subject of this comment.

Several authors have recently discussed systems that contain a time-dependent infinite solenoid [1-6]. In these works, the $z$ component of the magnetic field associated with this device is written in the following form:

$$B_z(r, t) = B(t)\theta(R - r) \tag{1}$$

where $r$ denotes the cylindrical radial coordinate, $R$ is the solenoidal radius and $\theta(x)$ is the unit step function which vanishes for a negative argument and is unity for a positive one. The solenoidal axis coincides with the $z$ axis of cylindrical coordinates. Units used in this work are those where the speed of light $c = 1$. All calculations are carried out for the vacuum and the fields $\boldsymbol{D}$ and $\boldsymbol{H}$ are omitted from the discussion. (Some authors use a related expression where the solenoidal radius shrinks to zero in a manner which conserves its magnetic flux.) The main discussion of the present work shows that (1) holds *only* if $B(t)$ is a linear function of the time.

Using Maxwell equations and the symmetries of the solenoid, one proves that $B_r = B_\varphi = 0$. Relying on this result, the following analysis shows the validity range of (1).

Assume that (1) holds. Consider a circle of radius $r > R$, which is a part of the $x, y$ plane and whose centre is at the origin. Using vector analysis, Maxwell's equations and (1), one finds for this circle

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\dot{B}(t)S \tag{2}$$

where $S$ denotes the area of the solenoidal cross section. The cylindrical symmetry of the problem shows that

$$E_\varphi = -\dot{B}(t)S/2\pi r. \tag{3}$$

Another Maxwell equation proves that at the outer region

$$(\text{curl}\,\boldsymbol{B})_\varphi = \frac{\partial E_\varphi}{\partial t} = -\ddot{B}(t)S/2\pi r. \tag{4}$$

Use of (3) and $\boldsymbol{E} = -\partial \boldsymbol{A}/\partial t$ shows that the potential can be written as follows:

$$A_\varphi(r, t) = B(t)S/2\pi r. \tag{5}$$

Now, if (1) holds, then using $B_r = B_\varphi = 0$ one finds that $\boldsymbol{B}$ vanishes identically at the cylindrical outer region. Hence, curl $\boldsymbol{B}$ must also vanish there. It follows from (4) that $\ddot{B}(t) = 0$. This result proves the main claim of this comment which says that if (1) holds then $B(t)$ must be a linear function of the time.

The following point is relevant to this result. Assume that a device uses an infinite solenoid whose flux is a linear function of the time and both (3) and (5) hold. Considering these expressions, the potential and the field at an outer point *look* like instantaneous functions of the time $t$ and are independent of the retarded time. The following argument shows that one cannot use this device for the purpose of violating causality by means of transmitting signals faster than light. Indeed, a signal emanating from the solenoid must deviate from the linear time dependence of $B(t)$. But the main result of this comment shows that if $B(t)$ does not satisfy linearity then (1), (3) and (5) do not hold. In this case one has to resort to the generally valid calculations of the retarded potentials [7]. This conclusion proves that the system is compatible with causality.

Another result is related to the relevance of a time-dependent solenoid to the magnetic Aharonov-Bohm (AB) effect [8, 9]. Classical equations of motion of a charge, i.e. the Lorentz force law, are written in terms of electromagnetic fields. On the other hand, in quantum mechanical equations, like the Schrödinger equation or the Dirac one, charge-field interactions are expressed by means of potentials. Considering these interactions, the correspondence between quantum mechanics and classical physics is shown in Ehrenfest's theorem [10]. The AB effect discusses the particular case of a charge which is confined to a field-free region of space where the potential is non-zero. In such an experiment, no classical force affects the motion of the charge. However, quantum mechanical calculations predict that, under certain circumstances, a non-zero phase shift should alter the interference pattern of the charge used in the AB experiment. Indeed, it turns out that there are cases where a phase shift of this kind is measured for an electron moving in a field-free region of space [11].

This discussion indicates that a solenoid can be used in the AB effect if its outer region is field free. Evidently, the results of this comment show that this requirement cannot be satisfied by a time-dependent solenoid. This claim can be deduced in the following way. Assume one builds a linearly time-dependent solenoid where, as shown above, the *magnetic* field vanishes at the solenoidal outer region. In this case, the *electric* field (3) is non-zero unless the solenoid is time independent. Thus, if one adheres to the original field-free requirement, then it is proved that a time-dependent solenoid cannot be used in an experiment designed for the measurement of the AB effect.

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